METHOD FOR OBTAINING A REGULAR STRUCTURE IN THE DEFORMABLE REGION OF A CONTINUUM

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In mechanics there has always been considerable interest in processes in which a structure arises by some means or other. Classic examples of these processes are formation of Bernard cells with convective flotation of a layer of liquid, Taylor vortices in shear flows between coaxial cylinders, formation of a regular system of vortices in stratified shear flow (the Kelvin–Helmholtz instability), etc. [1]. The mechanics of rocks and loose materials also has many similar interesting examples. Probably everyone has observed a set of polygonal cracks which forms when the earth dries after rain. Here a structural heirarchy may form when there is formation of large blocks with the same scale, then they divide into blocks with a smaller scale, etc. This situation is typical for rocks [2].

Regular structures may also arise in loose material. We take the simplest situation of uniform shear of a sample of loose material. In fact we are talking about Couette flow between parallel plates. It is possible to realize it in special equipment for uniform shear (in this connection it is necessary that in contrast to viscous liquids there are no attachment conditions at the boundary). With small shears flow is plane-parallel in nature, and then in the material a three-dimensional and quite ordered structure forms. Here the material is broken down into individual cells, and shear is localized at their boundaries [3].

Depending on material properties and loading conditions the structure may be three-dimensional, provisional, or of a mixed nature. Generally all flow processes for liquids and other more complex materials (here for brevity we also include processes of solid deformation) may be broken down into three major classes: 1) processes in which there is no structure formation; 2) processes with formation of a structure; 3) other processes. Of course this is not a strict classification, but here it is entirely satisfactory.

We pose the problem: is it possible to describe the second class of flow more or less constructively? It is clear that for flow itself its nature depends on the rheology of the medium (i.e., the material), and loading conditions and regimes (initial and boundary conditions, and also mass forces). These parameters may be referred to as controlling parameters connected with reaction of the material (energy dissipation rate, stress, and strain distribution, the fact itself of structure formation and its characteristics), are controlling. Then the problem is formulated in a different way: from the multitude of combinations of controlling parameters a class is separated which leads to formation of a structure.

As in [4] we consider structures not as statistical data, but as something which arise during evolution of a system, and in our case a deformable maternal—external conditions system. Let T be a scalar parameter which specifies the momentary state of the system (for example T may be the intensity of loading, shear, etc.) and it plays the role of a physical or some internal time. Always it is possible to normalize T so that the initial state corresponds to $T = 0$, development of the process to an increase in T, and formation of a structure to values from T^* to T^0.

In this situation the original problem may be written as follows: under what conditions should the process be satisfied with $T < T^*$. Or in other words from what flows and deformation processes may quite regular structures develop. This problem is simpler than the original one since it relates to description of some class of normal flows without structures (basic flows).

The regularity of a structure points to some invariance of it in space. In fact this leads to the situation that the distribution of stresses, strains, and local energy dissipation acquires a more or less periodic nature. Therefore, a process with a structure is to a certain extent uniform and spatially invariant. This view leads to the following idea:

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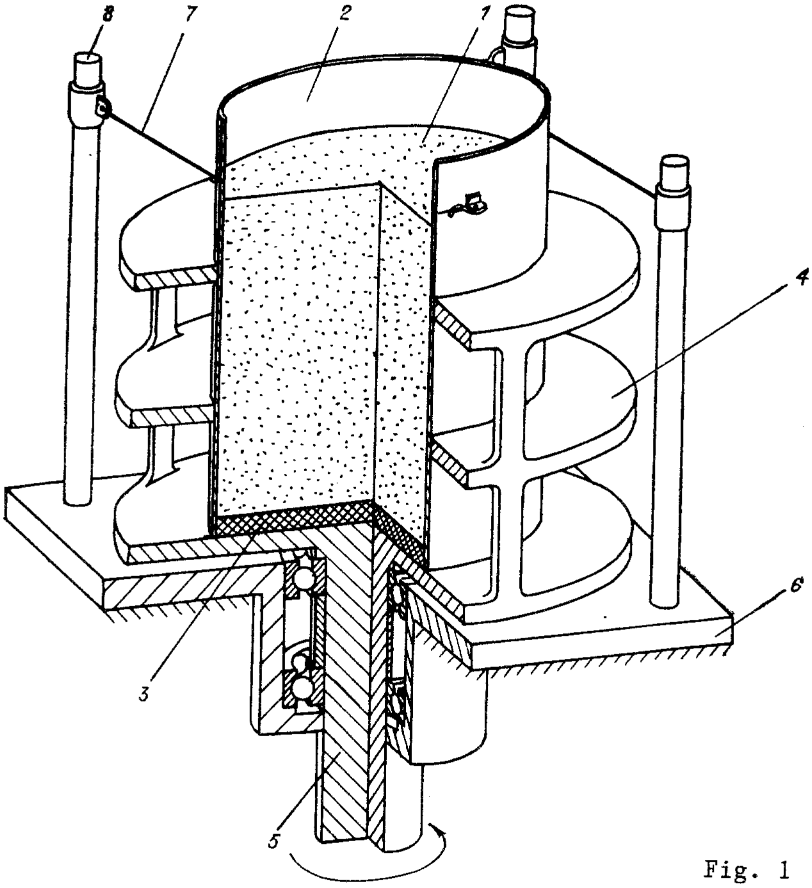

Fig. 1

flow which precedes occurrence of a structure should be as much as possible uniform with respect to space. In an ideal situation this is flow in which stresses and strains, and this means their rates, do not depend on spatial coordinates. This characteristic is sufficient for accurate description for the corresponding class of flows, and this was partly done in [5]. Of course, well-known flows (for example, Couette flow between parallel plates) fall into the class in [5]. It appears that in spite of the almost trivial nature of the statement of the problem new flows were detected here including those which it is entirely reasonable to realize.

Uniform flows were considered in [5] in connection with the possibility of using them for rheometric studies. There the most important thing was retention of flow uniformity, i.e., maintenance of those regimes when $T < T^*$. Here in a certain sense the problem is the opposite one, i.e., to create conditions when flow evolves beyond a limiting value T^*. Analysis is most simple if we revert to conditions of uniqueness. It is shown in [5] that uniform flow is unique if stability is retained for a deformable body and there is also rheological stability for the material; absence of inertial and other mass forces; the material is uniform. The latter will also be assumed below since structures connected with material inhomogeneity, and this means those in a hidden form from the start, are not of interest. Thus if the uniqueness conditions indicated above are set, it is possible to expect development of regular structures.

Let boundary conditions be prescribed at the boundary corresponding to uniform flow, and the loading regime lead T beyond the region $(0, T^*)$. In this case it is not indifferent to how conditions are prescribed at the boundary. We clarify this by a specific example. Let the basic deformation process be reduced to tension of a rod $0 \le x \le L$, $|y| \le h/2$ (x, y are Cartesian coordinates, L, h are rod length and width ($h << L$), plane strain). Displacement components have the form

$$u = \frac{u^0}{L} x, \quad v = -\nu \frac{u^0}{L} y \tag{1}$$

(u^0 is displacement of the end $x = L$, ν is a material characteristic). This process may be realized very simply; at the end of the rod $x = L$ a tensile force $P = P(u^0)$ is applied, but the side surfaces $|y| = +h/2$ are free from stresses. However, it is easy to see that in this

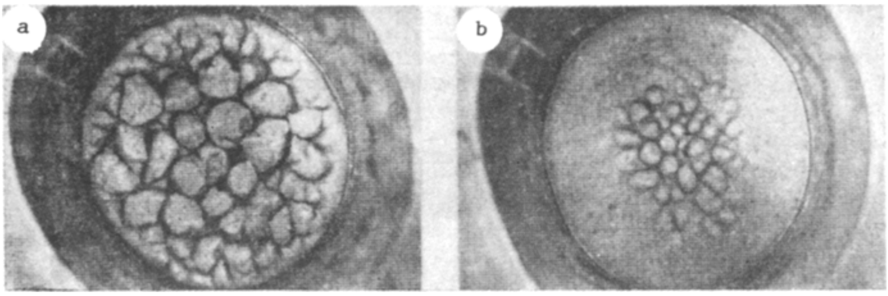

Fig. 2

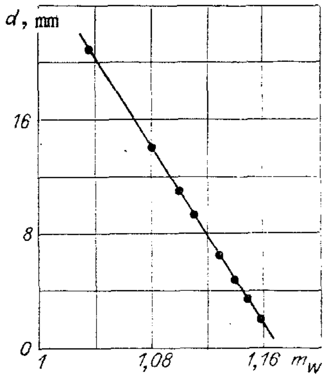

Fig. 3

case with transition through a critical condition as a rule there will be no structure whatsoever. For definiteness we assume that the critical state relates to transition of the curve P—u^0 into a descending branch. Then any disturbances (for example, the width h) leads to the situation that the rod only separates into two parts.

The same uniaxial tension may be realized in another way: by prescribing over the whole closed boundary x = 0, L, $|y| \leq h/2$ and $y = \pm h/2$, $0 \leq x \leq L$ for displacement vector (1). In the precritical state the result is the same as that with stresses prescribed at the boundary. However, with transition through T^* the situation is different in principle. Perturbations will not spread over the whole length and the rod is separated into parts by a quite regular crack system (a similar problem was considered in [6]). Thus, for our purposes conditions at the boundary should be as close as possible to the second type (stiff loading).

Thus, the method for realizing flows with a structure may be reduced to the following sequence of operations: 1) a specific base for uniform flow (for example, from the class in [5]) is chosen; 2) the original configuration of the deformable region is chosen and for it the corresponding boundary conditions are determined; 3) a loading device is created which realizes boundary conditions in the stiffest possible way; 4) material rheology, loading parameters, and possibly also boundary configuration (according to (2)) are chosen so that at a certain instant the uniqueness conditions of strain and stress distribution are upset.

Here it is appropriate to consider one generalization. It is possible in a natural way to broaden the class of basic flows by weakening some of the requirements for uniformity. For example, it is not necessary that strain in the basic flow depend on all three coordinates x, y, and z, but we say that it only depends on x and y. Then it is also not necessary for precise fulfillment of boundary conditions which follow from strain uniformity (especially as with precise realization of them considerable difficulties arise).

Thus, as a result of accomplishing this algorithm we obtain flow with a certain regular structure. In particular cases this is proved theoretically (see [6, 7]). In the general case apparently proof is impossible. It is sufficient to quote that known structures, and primarily methods for realizing them, are contained in this algorithm. By using this it is possible to obtain new forms of regular structures.

We consider a number of examples. As a basis we choose plane elliptical flow [5, 8]. This is superposition of a sequence of Couette flows between parallel plates. Total strain uniformity is provided if at the boundary of an elliptical region a Keplerian distribution of velocities is prescribed:$(\mathbf{v}\cdot\mathbf{n}) = 0$, $\mathbf{v} \times \mathbf{r} =$ const ($\mathbf{n}$ is the normal to the boundary, $\mathbf{r}$ is the radius-

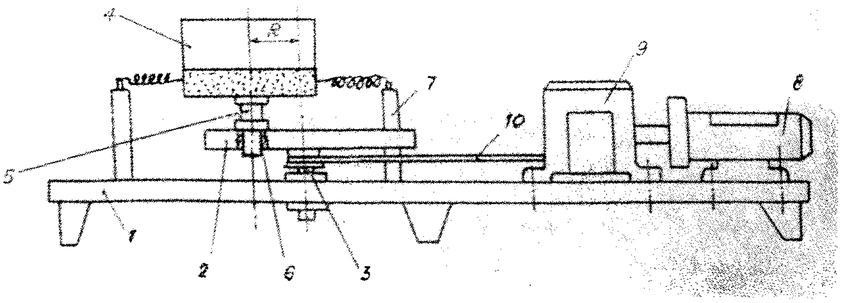

Fig. 4

Fig. 5

Fig. 6

vector from the center of the ellipse). For practical realization it is convenient to use generalization and the last boundary condition is replaced by one close to it: $|\mathbf{v}|$ = const. Thus, at the boundary of the elliptical region the following kinematic conditions should be fulfilled

$$(\mathbf{v}\cdot\mathbf{n}) = 0, \ |\mathbf{v}| = \text{const.} \tag{2}$$

A device for realizing (2) is shown in Fig. 1. A specimen of material 1 is placed in a deep cylindrical chamber 2. The side surface of the chamber is a shell of elastic thin sheet metal. The bottom of the chamber is closed uniformly by stretched rubber 3 fastened to the inner surface of the shell. The loading device is rigid plate-formers 4 with coaxial elliptical cut-outs. The cylindrical chamber is placed within them. Since the length of the elliptical cut-outs equals the length of the boundary of the chamber cross section, then after installation the chamber takes on the shape of an elliptical cylinder. The plates embrace the shell in two sections over the height and are fastened on shaft 5 through bearings installed in a base 6. Loading is performed by rotating the plates from an electric drive not shown in Fig. 1) with respect to the fixed chamber. The rotating friction moment which arises at the chamber surface is compensated by flexible ties 7 fastened to fixed uprights 8.

The degree of loading stiffness depends to a certain extent on the choice of material. We consider deformation of specimens of dry and water-impregnated sand. Then the condition $(\mathbf{v}\cdot\mathbf{n}) = 0$ is fulfilled, and $|\mathbf{v}|$ = const is in fact replaced by the condition of external dry friction $|\tau_n| \le -f\sigma_n$ (τ_n, σ_n are tangential and normal stresses, f is external friction coefficient). If the inequality is rigorous, then the condition $|\mathbf{v}|$ = const is precisely maintained, and if there is not equality then sliding along the shell is possible. On the whole the loading method suggested is quite stiff.

We turn to choice of material. Dry sand satisfies the requirements formulated above only in cases when significant shears are realized. In loading method (2) characteristic shear is estimated by the value $(1 - K)$ ($K = b/a$ is coefficient of ellipse compression, b, a are its semi-axes). Experiments show that with small $(1 - K)$ the deformation process has a smooth stable character and no structure arises whatsoever. Here packing of particles, stress distribution, and velocities move into a steady condition. With an increase in strains $(1 - K)$ and transition of them through a critical value the picture changes sharply. A system of slip lines forms in the material, then as a result of convective rotation it ceases to function, a new system forms, etc. These systems separate specific structural cells in the material [9].

TABLE 1

Storage conditions	Size of clods, mm			
	alumina	flour	gypsum	kaolin
Calcined	1,2	3	4	7,5
Exposure under room conditions	1,5	4,5	5,5	8
Exposure in a 'water bath'	2,5	6	7	11,5

The rheology of water-impregnated sand also satisfies the requirements indicated above. Compared with dry sand there is marked adhesion. Therefore critical strains are reduced, and moreover both slip lines and normal separation cracks become possible. This means that if in a uniform process critical strains are reached the uniformity is unavoidably upset.

Tests were performed in the following sequence. First, a specimen of dry sand was placed in the chamber and loading was carried to transition of packing into a steady state. Here parameters were selected so that no structure arose whatsoever ($K = 0.91$, $a = 60.8$ mm, $b = 55.3$ mm). Quartz sand was used with a particle size of 0.3 mm, specimen weight 545 g, and in the steady state the specimen volume $V = 320$ cm^3, pore volume $V_p = 115$ cm^3, and the porosity is 36%.

After reaching a steady state loading was ceased and liquid was introduced into the specimen. In all tests its volume V_L exceeded V_p, i.e., in the original state the impregnation factor $m_W = V_L/V_p > 1$. Therefore before deformation at the specimen surface a layer of of excess liquid always remained. Then loading was started for a water-impregnated specimen. With rotation of the plates packing underwent positive dilation, and therefore the layer of liquid was sucked from the specimen surface into the increased pore volume. On reaching critical strains for the specimen a system of parallel cracks forms. As a result of convective rotation cracks emerge from under load and a new system develops. In this respect the process is similar to deformation of dry sand. The difference consists of the fact that dry sand cracks are only shear in nature and they heal, but here due to adhesion a specimen separates into individual stable cells. The excess of liquid is concentrated at its boundaries. As a result of rotation the cell angles are smoothed and their shape becomes round (Fig. 2, a) $V_L = 125.5$ g, b) 131 g).

It can be seen in phogographs that apart from a characteristic average size cells are also encountered with small dimensions obtained mainly with chambering of the initial cell angles, and that an increase in liquid leads to a reduction in average cell dimension d, and the dependence on coordinates average size – impregnation factor is almost linear (Fig. 3).

Above basic flow is realized by describing special displacements at the boundary. In principle it is possible to create them also by special mass forces. We consider one example. Let the specimen be a horizontal layer $0 \leq z \leq h$ placed on a backing $z = 0$. We shall displace the backing in its plane over a circle of small radius R. Then in each layer of the element specific forces of inertia start to operate. If the role of the side walls is removed, then immediately it can be seen that the stress-strained state of the layer may only depend on coordinate z and on time t, i.e. there is no dependence x and y. Therefore, this deformation may be referred to as basic.

Powder materials were used in the tests. Shown in Fig. 4 is the device for realizing this loading scheme. A disk 2 with a shift 3 is fastened to a base 1. A layer of powder is placed in a cylindrical cup 4 with a diameter of 120 mm. A series of holes 6 is arranged in the disk at different distances from its center. In one of them through a bearing a shaft 5 is held to which the cup is attached. Uprights 7 hold the cup through springs with rotation of the disk. The disk drive is accomplished by a motor 8 through a reducer 9 and a belt transmission 10. The device is fitted with a rheostat by means of which it is possible to change the motor revolution smoothly [10].

The scale of forces of inertia with loading intensity is determined by the value $\lambda = R\omega^2$ (ω is angular velocity of disk rotation). In tests radius R was fixed, but ω was gradually increased. As might be expected, with small λ the process is stable in nature.

With transition through a critical value structural elements form, shown in Fig. 5. The mechanism of their formation is connected with surface instability of the layer. Particles at the free surface are drawn into movement as a result of adhesion with the lower layer and the side action of upper layer particles. With high forces of inertia adhesion is overcome and particles fall behind movement of the backing. At this time it is in contact with different particles of the backing and it 'interrogates' them on the subject of possible attachment. If the material exhibits sufficient sticking, then the 'interrogation' leads to formation of clods. Shown in Fig. 5 are clods with tests on kaolin ($R = 5$ mm, $\omega = 18.8$ $\sec^{-1}$). It can be seen that apart from large spherical formations there are also simultaneously small rudimentary formations. The time of their growth to critical dimensions (when they start to break) is proportional to the rolling path over the underlying layer. The critical size itself is determined by forces of particle adhesion tending to maintain their spherical shape.

Thus, from the size of the clods it is possible to assess the tendency of material towards forming agglomerates, and consequently to caking. All of these indices depend on powder moisture content, and therefore it may be estimated indirectly. Given in Table 1 are data for the average dimensions of clods for different materials. Treatment of the results is somewhat complicated by the fact that the process of clod formation is dynamic in nature. During formation they start to react with each other, they break, and then grow again, etc. Therefore in tests the rotation rate is gradually increased at first to a certain limit, and then it is reduced to zero. After this treatment of the results was carried out.

In this method directions x and y are the same as any in this plane, and they are entirely of equal value. Such are the specific loading conditions.

Now we consider loading with a separate direction x. Let a layer of material contain reciprocating motion in its plane in direction x. Here with a certain frequency there is also surface loss of stability and a structure arises with a characteristic size along axis x (Fig. 6, water-impregnated sand, amplitude 3 mm, frequency 10 Hz).

Thus, known methods for realizing dissipative structures are refined in the suggested algorithm. The algorithm makes it possible to obtain new dissipative structures. In a number of cases they are of direct practical interest.

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